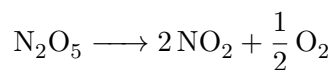


Exercise 7

Experiments show that if the chemical reaction



takes place at 45°C , the rate of reaction of dinitrogen pentoxide is proportional to its concentration as follows:

$$-\frac{d[\text{N}_2\text{O}_5]}{dt} = 0.0005[\text{N}_2\text{O}_5]$$

(See Example 3.7.4.)

- Find an expression for the concentration $[\text{N}_2\text{O}_5]$ after t seconds if the initial concentration is C .
- How long will the reaction take to reduce the concentration of N_2O_5 to 90% of its original value?

Solution

Part (a)

Multiply both sides by -1 .

$$\frac{d[\text{N}_2\text{O}_5]}{dt} = -0.0005[\text{N}_2\text{O}_5]$$

Divide both sides by $[\text{N}_2\text{O}_5]$.

$$\frac{1}{[\text{N}_2\text{O}_5]} \frac{d[\text{N}_2\text{O}_5]}{dt} = -0.0005$$

Rewrite the left side using the chain rule.

$$\frac{d}{dt} \ln[\text{N}_2\text{O}_5] = -0.0005$$

The function you take a derivative of to get -0.0005 is $-0.0005t + D$, where D is any constant.

$$\ln[\text{N}_2\text{O}_5] = -0.0005t + D$$

Exponentiate both sides to get $[\text{N}_2\text{O}_5]$.

$$e^{\ln[\text{N}_2\text{O}_5]} = e^{-0.0005t + D}$$

$$[\text{N}_2\text{O}_5] = e^D e^{-0.0005t}$$

Use the fact that the initial concentration is C to determine e^D .

$$[\text{N}_2\text{O}_5](0) = e^D e^{-0.0005(0)} = C \quad \rightarrow \quad e^D = C$$

Therefore, the concentration is

$$[\text{N}_2\text{O}_5](t) = C e^{-0.0005t}.$$

Part (b)

In order to find how long it'll take the initial concentration of N_2O_5 to reduce to 90% of its original value, set $[\text{N}_2\text{O}_5](t) = 0.9C$ and solve the equation for t .

$$[\text{N}_2\text{O}_5](t) = 0.9C$$

$$Ce^{-0.0005t} = 0.9C$$

$$e^{-0.0005t} = 0.9$$

$$\ln e^{-0.0005t} = \ln 0.9$$

$$-0.0005t = \ln 0.9$$

$$t = -\frac{\ln 0.9}{0.0005} \approx 210.721 \text{ seconds}$$