Exercise 7

Experiments show that if the chemical reaction

$$N_2O_5 \longrightarrow 2\,NO_2 + \frac{1}{2}\,O_2$$

takes place at 45°C, the rate of reaction of dinitrogen pentoxide is proportional to its concentration as follows:

$$-\frac{d[N_2O_5]}{dt} = 0.0005[N_2O_5]$$

(See Example 3.7.4.)

(a) Find an expression for the concentration $[N_2O_5]$ after t seconds if the initial concentration is C.

(b) How long will the reaction take to reduce the concentration of N_2O_5 to 90% of its original value?

Solution

Part (a)

Multiply both sides by -1.

$$\frac{d[N_2O_5]}{dt} = -0.0005[N_2O_5]$$

Divide both sides by $[N_2O_5]$.

$$\frac{1}{[N_2O_5]}\frac{d[N_2O_5]}{dt} = -0.0005$$

Rewrite the left side using the chain rule.

$$\frac{d}{dt}\ln[N_2O_5] = -0.0005$$

The function you take a derivative of to get -0.0005 is $-0.0005t + D$, where D is any constant.

$$\ln[N_2O_5] = -0.0005t + D$$

Exponentiate both sides to get $[N_2O_5]$.

$$e^{\ln[N_2O_5]} = e^{-0.0005t+D}$$

$$[N_2O_5] = e^D e^{-0.0005t}$$

Use the fact that the initial concentration is C to determine e^D.

$$[N_2O_5](0) = e^D e^{-0.0005(0)} = C \quad \rightarrow \quad e^D = C$$

Therefore, the concentration is

$$[N_2O_5](t) = Ce^{-0.0005t}.$$

Part (b)

In order to find how long it'll take the initial concentration of N_2O_5 to reduce to 90% of its original value, set $[N_2O_5](t) = 0.9C$ and solve the equation for t.

$$[\mathrm{N_2O_5}](t) = 0.9C$$

$$Ce^{-0.0005t} = 0.9C$$

$$e^{-0.0005t} = 0.9$$

$$\ln e^{-0.0005t} = \ln 0.9$$

$$-0.0005t = \ln 0.9$$

$$t = -\frac{\ln 0.9}{0.0005} \approx 210.721 \text{ seconds}$$